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"Influence of Measurement Technique on the Air-Void Structure of Hardened Concrete."

Paper by Tine Aarre.

Discussion by Ken Snyder.

Introduction

The author proposes, and successfully demonstrates, that the results for air content and specific surface area (SSA) from an ASTM C 457 linear traverse performed on a thin section can differ from those performed on a polished plane section. The difference between thin section and plane surface air contents was quantified using a probability theory approach, and semi-quantitative arguments were given to explain the difference for SSA results. However, the tabulated data suggest further study is needed. This Discussion advances the author's work by addressing a minor error in the calculation, and by proposing an analytic approach for estimating the influence of measurement technique on the measured SSA value.

Air Content

The error in the air content calculation was discovered while studying the author's Fig. 8 which was a plot of the function $K(R,t)$ that converts an observed thin section air content to the actual air content for a characteristic sphere radius R and thin section thickness t. For a system of monosized spheres, if the diameter of the sphere $(2R)$ is less than the thin section thickness (t) there can be no observed circle, the expected area of a circle becomes zero, and the correction factor should diverge to infinity at $2R = t$. Contrary to this, the author's curve is continuous at $2R = t$, and has a value of zero at $2R = 0$.

The cause for this error occurs in the author's Eqn. 4 which gives the expected area of a circle seen on a thin section. As demonstrated, the expected area of an apparent circle can be expressed as a sum of integrals over three intervals: 1) no observable circle, 2) the apparent surface is the bottom surface, and 3) the apparent surface is the top surface. However, the bottom surface remains the apparent surface until $x = t/2$, not $x = 0$ as in the author's text. Only for $x \geq t/2$ is the top surface the apparent surface. Therefore, the second integral in Eqn. 4 should be over the interval $[-R+t, t/2]$, and the third integral over the interval $[t/2, R]$. Further, using symmetry arguments, the entire quantity in Eqn. 4 can be equivalently expressed as

$$\begin{aligned} A_a &= \frac{1}{R}\int_{t/2}^{R}(R^2 - x^2)\pi\, dx \\ &= \frac{2}{3}\pi R^2\left[1 - \frac{3}{2}\left(\frac{t}{2R}\right) + \frac{1}{2}\left(\frac{t}{2R}\right)^3\right] \end{aligned} \tag{1}$$

This result agrees with the analytic prediction derived elsewhere [1]. As defined by the author, the conversion factor K(R,t) is the ratio of the true plane and the apparent air contents:

$$K(R,t) = \frac{A_{tr}}{A_a} = \frac{1}{1 - \frac{3}{2}\left(\frac{t}{2R}\right) + \frac{1}{2}\left(\frac{t}{2R}\right)^3} \tag{2}$$

This corrected value for $K(R,t)$ is plotted here in Fig. 1 along with the author's original equation. To generalize the results, the correction factors are plotted as a function of $2R/t$. The dotted line represents the point at which the conversion factor for the air content should diverge to infinity. It would appear as though the correction to the author's equation may be insignificant. However, an entrained air void system can be modelled reasonably well by a lognormal sphere diameter distribution with a modal diameter of 30 μm. For the author's example with $t = 20$ μm, the correction at the modal diameter is appreciable.

Specific Surface Area

For the case of specific surface area, the author used dimensional arguments to explain the behavior between the planar and the thin section analysis results. A quantitative correction factor for the SSA can be derived from the relationship between statistical moments of the sphere radii distribution and the circle radii distribution. Let the notation $\langle Y^n \rangle$ represent the n-th statistical moment of the circle radii distribution observed either on the plane or from the thin section. The SSA can be expressed as a ratio of circle radii moments[2]:

$$\alpha_{tr} = \frac{32}{\pi}\frac{\langle Y_{tr} \rangle}{\langle Y_{tr}^2 \rangle} \qquad \alpha_a = \frac{32}{\pi}\frac{\langle Y_a \rangle}{\langle Y_a^2 \rangle} \tag{3}$$

The second moment is proportional to the expected area of a circle:

$$\langle Y_{tr}^2 \rangle = \frac{1}{R}\int_0^R R^2 - x^2 dx = \frac{2}{3}R^2 \tag{4}$$

$$\begin{aligned} \langle Y_a^2 \rangle &= \frac{1}{R}\int_{t/2}^R R^2 - x^2 dx \\ &= \langle Y_{tr}^2 \rangle \left[1 - \frac{3}{2}\left(\frac{t}{2R}\right) + \frac{1}{2}\left(\frac{t}{2R}\right)^3\right] \end{aligned} \tag{5}$$

The first moment of the circle radii distribution for the true plane value and the apparent thin section value are calculated in an analogous manner:

$$\langle Y_{tr} \rangle = \frac{1}{R}\int_0^R \sqrt{R^2 - x^2} dx = \frac{\pi}{4}R \tag{6}$$

$$\begin{aligned} \langle Y_a \rangle &= \frac{1}{R}\int_{t/2}^R \sqrt{R^2 - x^2} dx \\ &= \langle Y_{tr} \rangle \left[1 - \frac{2}{\pi}\left(\frac{t}{2R}\right)\sqrt{1 - \left(\frac{t}{2R}\right)^2} - \frac{2}{\pi}\sin^{-1}\left(\frac{t}{2R}\right)\right] \end{aligned} \tag{7}$$

Combining all of these results gives

$$\alpha_a = \alpha_{tr} \frac{\left[1 - \frac{2}{\pi}\left(\frac{t}{2R}\right)\sqrt{1 - \left(\frac{t}{2R}\right)^2} - \frac{2}{\pi}\sin^{-1}\left(\frac{t}{2R}\right)\right]}{\left[1 - \frac{3}{2}\left(\frac{t}{2R}\right) + \frac{1}{2}\left(\frac{t}{2R}\right)^3\right]} \tag{8}$$

$$= \frac{\alpha_{tr}}{L(R,t)} \tag{9}$$

where $L(R,t)$ is defined here as the corresponding correction factor for SSA. The correction factor $L(R,t)$ is also plotted here in Fig. 1 as a function of $2R/t$. Note that the value of $L(R,t)$ is always less than one, and that it approaches unity "faster" than $K(R,t)$. This behavior of $L(R,t)$ may explain why many of the observed values of α from the thin section data were nearly equal to, or greater than, those from the planar analysis.

Air Void Radii Distribution

The final solution to calculating a correction factor for either total air content or specific surface area can only be achieved after averaging these correction factors over the sphere radii distribution. The distribution of air void radii in air-entrained concrete spans more than two orders of magnitude. Although the author has obtained qualitative estimates of correction factors approximating the air void system as a monosized distribution, better corrections might be obtained from numerical averages of the correction factors.

The evidence for this argument lies in the author's published data for planar and thin section analysis from 10 different mixes. From each mix the air content and the SSA, along with coefficients of variation, are calculated for each method. The author averages the results from the planar analysis and the thin section analysis separately, and then calculates the ratio of these averages. This approach lacks a physical basis because the behavior among the results for a single measurement for different mixes should be independent (*i.e.*, the thin section air content of Mix 1 should be independent of the thin section air content of Mix 2, etc.). However, the author argues in the text for a common ratio among mixes. Therefore, the author should have calculated the ratio for each mix, and averaged the resulting ratios. The results of these calculations are reported here in

Table 1. The estimated errors for each mix are calculated using the author's published data and the propagation of errors, a method which is described in detail elsewhere [3]. The quantities labeled $\bar{x}$ and s are the average and estimated standard deviation, respectively, of these 10 ratios. Although the average of the ratios for the air content is nearly equal to the author's ratio of averages, an examination of the individual ratios is quite revealing. In most cases the uncertainty in the individual ratios was relatively large. It would appear as though the factor of 0.85 for air content is good in the mean, but would not be acceptable for individual cases. Further, the results for α are quite scattered, suggesting that a global average correction factor may not be appropriate, and that for both air content and SSA, more accurate estimates of the correction factor can only be obtained from averaging the correction factors over the sphere radii distribution.

Summary

The author has gone a long way towards demonstrating that the results from thin section analysis require careful analysis. Although the measured air content based on thin section data will be less than from planar analysis, estimating the effect of measurement technique on the specific surface area is more difficult. It would appear that the next step would be to average these quantities using parametric estimates of the sphere radii distribution using the planar probe chord distribution, coupled with Monte Carlo calculations for verification. With further study, accurate correction factors could be calculated in a self-consistent manner from the thin section data alone.

References

[1] Lu, B., and Torquato, S., *J. Chem. Phys.*, V. 98, 1993, pp. 6472–6482.

[2] Wicksell, S.D., *Biometrika*, V. 17, 1925, pp. 84–99.

[3] Wilson, E. Bright, *An Introduction to Scientific Research*, Ch. 9, McGraw-Hill, 1952.

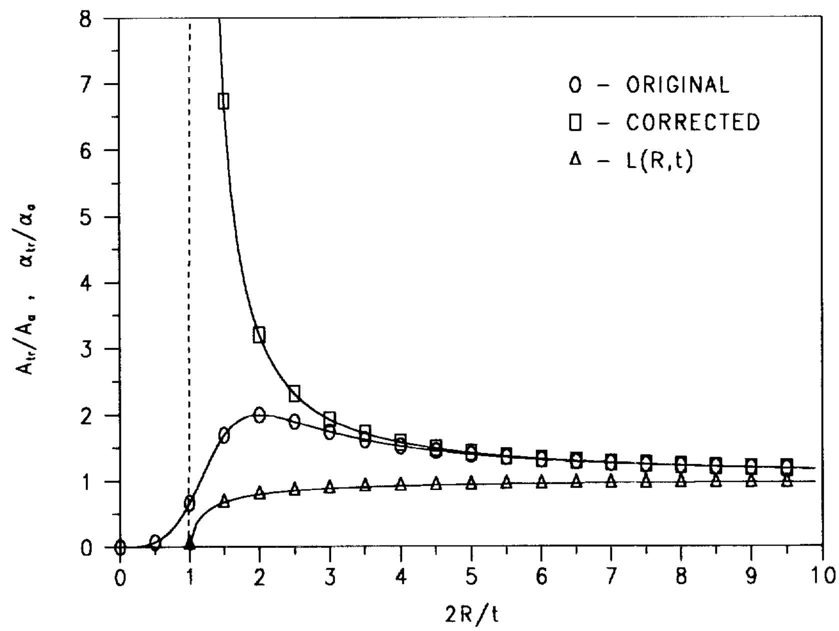

Figure 1: The original and the corrected air content correction factor, $K(R,t)$, and the specific surface area correction factor, $L(R,t)$ which is derived in the text.

Mix No.	A_a/A_{tr}	α_a/α_{tr}
1	$0.82 \pm .19$	$0.96 \pm .28$
2	$0.92 \pm .04$	$0.97 \pm .10$
3	$0.71 \pm .16$	$0.91 \pm .03$
4	$0.88 \pm .21$	$0.96 \pm .10$
5	$1.00 \pm .22$	$0.97 \pm .17$
6	$0.84 \pm .28$	$1.23 \pm .20$
7	$0.81 \pm .19$	$1.12 \pm .14$
8	$0.82 \pm .16$	$0.94 \pm .11$
9	$0.90 \pm .15$	$0.92 \pm .05$
10	$0.86 \pm .07$	$0.96 \pm .19$
	$\bar{x} = 0.86$ $s = 0.08$	$\bar{x} = 0.99$ $s = 0.10$

Table 1: The author's original data expressed as ratios. The average ($\bar{x}$) and the estimated standard deviation (s) for the ten ratios are given at the bottom of the column.